Prediction of crack growth between two neighboring circular holes under internal pressure in brittle material by numerical modeling

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ABSTRACT: Application of Numerical modeling of cracks has been very useful in solids. Finite Element Method has been applied to determine of first crack locations in special geometry which it can be use for rock fracturing. After appearance of the first cracks, its growing in the rock is very important to predict of fracture process. Length of the cracks is dependent to tangential stress due to NEEM’s pressure and tensile strength of the rock. Numerical modeling showed location of the first cracks is in the place that stress concentration be high. Two experimental cases were studied in this work. Once was rock fracture process between two neighboring holes on a granite mine and other was crack growth process in Brazilian tensile test (granite). Result of numerical simulations by finite element method showed size of elements near the crack tip must be equal to grain size of rock material. The results showed there is a good agreement between experimental works and numerical models.

Keywords: Finite element method, Rock fracturing, crack growth.

1. Introduction

One of the main methods in quarry mining, especially in hard rocks, is the controlled fracture method that is carried out by the introduction of a slowly advancing crack by Non-Explosive Expansion Material (NEEM). The application of NEEM in hard rock quarry mining has recently been increased (Hayashi, et al., 1994; Pal Roy, 2005; Arshadnejad, 2007). This method of rock breakage is without noise and vibrations and its operation, compared to blasting method, is more controllable, very safe and easy and without extra undesirable cracks in the rock block.

In this method, some circular holes are drilled closely with equal length, diameter and spacing (centre-to-centre distance) in a rock block. Subsequently, the holes are filled with the NEEM, which by its expansion will generate an incremental static load into the holes after about two to four hours (Goto, et al., 1988; Zhongzhe, et al., 1988). If the spacing of the holes is suitable, it will create a crack between two neighboring holes, and the rock will fracture along the high-stress concentration path between the holes. If the material of the medium is brittle, such as hard rocks (e.g. granite and

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quartzite), it will not yield and no plastic behavior before failure will be observed in the material (Jana, 1991; Hoek & Bieniawski, 1965; Lajtai, 1972; Ingraffea & Schmidt, 1978). Thus, the material is considered to behave in a linear elastic mode until the onset of failure.

The brittleness index ($BI_{m}$) is introduced from the punch penetration test and is computed by equation as follow (Szwedzicki, 1998):

$$BI_{m} = \frac{F_{\text{max}}}{P}$$ (1)

Where $F_{\text{max}}$ is maximum applied force on a rock sample in kN, $P$ is the corresponding penetration at maximum force in mm. Table 1 shows the classification of rock brittleness was suggested based on the punch penetration test (Szwedzicki, 1998).

<table>
<thead>
<tr>
<th>Britteness index (kN/mm)</th>
<th>Britleness class</th>
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</thead>
<tbody>
<tr>
<td>≥ 40</td>
<td>Very high brittle</td>
</tr>
<tr>
<td>35-39</td>
<td>High brittle</td>
</tr>
<tr>
<td>30-34</td>
<td>Medium brittle</td>
</tr>
<tr>
<td>25-29</td>
<td>Moderate brittle</td>
</tr>
<tr>
<td>20-24</td>
<td>Low brittle</td>
</tr>
<tr>
<td>≤ 19</td>
<td>No-brittle (Ductile)</td>
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</tbody>
</table>

Most of granites are high to very high brittle. Because of brittleness index of them are more than 35 (Yağız, 2006; Yağız, 2009). Then stress distribution in the granite (in rock breakage) is in elastic condition. Yağız have suggested an empirical model for brittleness index as follow:

$$BI_{P} = 0.198\sigma_{c} - 2.174\sigma_{t} + 0.913\gamma - 3.807$$ (2)

Where $BI_{P}$ is rock brittleness in kN/mm based on empirical method. $\sigma_{c}$ and $\sigma_{t}$ are uniaxial compressive and tensile strength of rock in MPa, respectively. $\gamma$ is volumetric weight of rock in kN/m$^3$.

When there are two neighboring holes in a plate loaded internally, stress concentration will occur. The maximum elastic stresses (stress concentration) were examined by several methods, such as photoelastic analysis (Hoek & Bieniawski, 1963; Joussineau, et al., 2003), direct strain measurement (Nesetova & Lajtai, 1978; Chong, 1987) and numerical modeling (Bazant, 1982; Yan, 2007). There are many empirical models for estimating stress concentration in different geometry, such as a circular hole. Stress concentration factors around a circular single hole due to uniform and axisymmetrical external pressure were analyzed by Howland (1929), Frocht (1935), Lipson and Juvinall (1963). One of the first studies on plane elasticity in bipolar coordinates in an infinite plate with two circular holes was done by Jeffrey (1920).
Howland (1935) investigated the stress distribution around an infinite row of equal size circular holes spaced equally in an infinite elastic plate. The plate was subjected to a uniaxial stress field. Howland and Knight (1939) presented stress functions for the problems involving equal size circular holes. Ling (1948) developed a solution (in bipolar coordinates) for the stresses in a plate containing two equal circular holes when variable are distance between them. He considered three stress fields: uniaxial stress parallel and perpendicular to the line of centers and equal stresses in all directions (Gerçek, 2005). Haddon (1967), using the conformal mapping and complex variable techniques, presented a solution for stresses around two unequal circular holes in an infinite plate. The plate was subjected to a uniaxial stress field with a variable inclination to the line of holes centers (Gerçek, 2005). Obert and Duvall (1967) have studied the stress distribution around pillars (rock columns) between two parallel circular excavations subjected to uniaxial compressive external loading by photoelasticity method. If the type of external loading is tensile in biaxial, two empirical models were developed by Schulz (1942) and Peterson (1974). Ling and Tsai (1969) presented an analytical solution for the stresses in a thick plate of infinite size containing a spherical inclusion or cavity eccentrically located between the surfaces. The plate had subjected to a stress system symmetrical about the axis of revolution of the plate while the surfaces were stress-free. Gerçek (1988 & 1996) presented a solution for boundary stresses for two parallel circular tunnels located in a biaxial in situ stress field. It was obtained by superposing the solutions developed by Ling (1948). Zimmerman (1988 & 1991) suggested approximate equations for stress concentrations in an infinite elastic plate containing two circular holes.

2. Stress distribution

Stress distribution around a circular hole depends on the stress field condition. Kirsch (1898) initially studied this problem for a single circular hole under biaxial stress field. The stress distribution within a thick-walled cylinder under uniform external and internal loading is as follows (Timoshenko & Goodier, 1951).

\[
\sigma_r = \frac{(a^2 P_i - b^2 P_o)}{b^2 - a^2} + \frac{a^2 b^2 (P_i - P_o)}{r^2 (b^2 - a^2)}
\]

\[
\sigma_\theta = \frac{(a^2 P_i - b^2 P_o)}{b^2 - a^2} - \frac{a^2 b^2 (P_i - P_o)}{r^2 (b^2 - a^2)}
\]

Where \(\sigma_r\) and \(\sigma_\theta\) are the radial and tangential stresses, respectively, and \(r\) is the radial distance of the considered point from hole centre. \(P_i\) and \(P_o\) are internal and external pressures, respectively, and \(a\) and \(b\) are internal and external radius of thick-walled cylinder, respectively. Because of axisymmetry in the loadings and body geometry, there is no shear stress in the medium. As a convention in rock mechanics, the tensile stress is considered negative and the compressive stress is considered positive (Hoek & Brown, 1980; Goodman, 1989). The constraint for using thick-walled cylinder Equations is as follow (Shigley, 1956; Hertzberg, 1996).
\[ \frac{b-a}{a} > \frac{1}{20} \]  

If there is no external pressure \((P_o = 0)\) the Equations become:

\[ \sigma_r = \frac{a^2 P}{b^2 - a^2} \left( 1 + \frac{b^2}{r^2} \right) \]  

\[ \sigma_\theta = \frac{a^2 P}{b^2 - a^2} \left( 1 - \frac{b^2}{r^2} \right) \]  

If the thickness of the cylinder wall increases to infinite \((b \rightarrow \infty)\), the cylinder will transform to a circular hole in an infinite plate, such as a hole in a rock medium. Then, Equations 6 and 7 convert to:

\[ \lim_{b \to \infty} \sigma_r = \frac{a^2 P}{r^2} = P \left( \frac{a}{r} \right)^2 \]  

\[ \lim_{b \to \infty} \sigma_\theta = -\frac{a^2 P}{r^2} = -P \left( \frac{a}{r} \right)^2 \]  

When two or more circular holes in a plate are loaded by internal pressure, stress concentration will occur among those. When the stress intensity is equal to the rock fracture toughness, cracks may be initiated. Subsequently, the crack will grow however, as the length of the crack increases, the stress on the crack tip decreases, due to distancing from the hole, thus decreasing the stress concentration. Nevertheless, by increasing the stress induced from the hole due to application of NEEM, in due time, the stress intensity on the crack tip will again increase up to the level of rock fracture toughness. Thus, again the crack will grow farther, and this circle of events will repeat; hence, a controllable mechanism for crack growth may be achieved.

Arshadnejad and his colleagues (2009) have suggested two empirical models (by numerical modeling) for determining stresses distribution including stress concentration factors between two neighboring circular holes due to internal pressure of the NEEM are as follow. Validations of the equations are only for the points located on the line passing through the centers of the holes.

\[ \sigma_r = 1.0352 \left( \frac{d}{S} \right)^{0.001} \nu^{0.015} P_i \left( \frac{d}{2r} \right)^2 + \left[ 17390r^3 - 2569.5r^2 + 163.62r - 2.6522 \right] \]  

\[ \sigma_\theta = -1.1715 \left( \frac{d}{S} \right)^{0.124} \nu^{-0.025} P_i \left( \frac{d}{2r} \right)^2 + \left[ 60397r^3 - 7878.2r^2 + 351.67r - 5.1152 \right] \]  

Where r (from hole centre) and Pi (NEEM pressure), σ_r and σ_θ (MPa), ν is Poisson’s ratio of rock, d is the diameter of holes (m) and S is the edge-to-edge distance between two neighboring holes (hole spacing) (m).

3. Birth of first crack

As was stated, when two or more neighboring circular holes in a plate are loaded by internal pressure, stress concentration will occur among those. If stress value goes high to tensile strength of the rock, first crack will born on the line passing through the centers of the holes in both sides of the holes. Figure 1 shows a graph which states tangential stress distribution. The graph helps to evaluate of crack length. Left side of point M tangential stress is higher than tensile strength of rock, and then rock will fracture. Size of the crack is equal to arrow of OA.

![Fig. 1. Tangential stress and crack length evaluation](image)

4. Crack growth by numerical modeling

In this study, the ABAQUS computer code based on finite element method (FEM) was used for determining of stress distribution on the crack tip and evaluating of stress intensity factor in mode one (K_I) and Phase^2 computer code (Rocscience, 1999 & 2001) (FEM) was used to determine the crack growth between two neighboring holes by numerical analysis. In this respect, six nodal triangular elements with nodal averages were utilized (Phase^2). The model geometry and the parameters were selected based on real conditions of quarry mining operations. The internal pressures in the holes induce by expansion of non-explosive expansion material (NEEM). Figure 2 and 3 show the stress distribution on the crack tip. Figure 4 and 5 show displacement around the holes and the cracks. These figures have obtained by ABAQUS computer code, when NEEM’s pressure increases step by step.
Fig. 2. Maximum stress in plane strain condition (Step 1)

Fig. 3. Maximum stress in plane strain condition (Step 2)

Fig. 4. Displacement in plane strain condition (Step 1)
Stress analysis for crack propagation can be carried out using available fracture mechanics software (e.g. ABAQUS). It is also possible with a usual finite element program (e.g. Phase²) (Diederichs, et al., 2004). A crack tip is modeled using discontinuity elements (tension-free) within an isoparametric six-noded triangular FEM mesh. Mesh density on crack tip and its around is higher than other points which they have located far from crack and crack tip. Last version of Hoek-Brown criterion (2002) is employed and the loads are increased incrementally for each model. At each stage the crack is extended according to the zone of new tensile rupture indicated by the FEM elements. After that another model is made by the new extended length and orientation on the crack tip and increased load. Numerical modelings were shown location of the first cracks is in the place that stress concentration be high.

Another observation was in Brazilian tensile strength of rock. In this work birth and growing of a crack in granite of Natanz was studied. The samples were provided and tested with method of Brazilian test (ISRM suggestion) and recorded of crack growth process in several stages by high speed photography. Figure 6 shows the process of crack growth in the granite sample (First and secondary cracks). Numerical modeling showed a similar crack path like as figure 6 by FEM. The result of crack growth is summarized in figures 7 to 9.
Fig. 6. Crack growth’s process at Brazilian test in granite of Nataz

Fig. 7. Numerical modeling and crack propagation in Brazilian tensile test (Stages 1 & 2)

Fig. 8. Numerical modeling and crack propagation in Brazilian tensile test (Stages 3 & 4)
5. Conclusions

Design of hole pattern in quarry mining is very important. If two neighboring holes are close together, stress concentration will occur and it will cause to first cracks generate. Length of the cracks is dependent to tangential stress due to NEEM’s pressure and tensile strength of the rock. Numerical modeling showed location of the first cracks is in the place that stress concentration be high. Two experimental cases were studied in this work. Once was rock fracture process between two neighboring holes on a granite mine and other was crack growth process in Brazilian tensile test (granite). Result of numerical simulations by finite element method showed size of elements near the crack tip must be equal to grain size of rock material. The results showed there is a good agreement between experimental works and numerical models. Authors think that there may be relation between brittleness index and fractal dimension of crack path. This can be a subject for future research.

References


